

Final Project Info

1 Summary

Groups of 2-3 students will research a topic that builds on the topics covered in class. The topic will be chosen in consultation with me. Your group will write a 4 – 8 page paper on your research topic and give a presentation to the class on your research. This project gives you a chance to explore an area you find interesting related to the class.

The project will need to have:

- a significant mathematical component such as proofs or computations and
- have good exposition (in other words, the paper needs to be written well enough that other students can learn something from the paper and the same for the presentation).

The target audience for both the paper and the presentation is other students in the class.

Your grade on the final project will be determined by:

- 5% Consulting with me to choose group and topic
- 5% Abstract with resources
- 15% Draft of paper
- 15% Constructive feedback on 2 other groups' papers
- 40% Final draft of paper
- 20% Presentation

Important Dates:

Jan 14 – First day of classes

Feb 5 – Topic and group chosen for final project

Feb 26 – Abstract for final project due

Mar 26 – Draft of final project paper due

Apr 9 – Feedback on other students' papers due

Apr 27 – Last day of class / Final project paper due before class

2 Choosing a group/topic

In consultation with me, you will need to choose a group of 2-3 people and a topic for your final project by Feb 5th. Possible topics can be found, for instance, in the textbook or the book, *Office Hours with a Geometric Group Theorist*. To help you find other people interested in topics you are interested in, we will

use Piazza to discuss possible topics. During this process, your group should talk to me in office hours about your topic idea to flesh it out. Topic choices will be submitted on Gradescope.

3 Abstract

A two to three paragraph description of what your group plans on researching with 2-3 resources (or more if you want them). The abstract is due Feb 26, and submitted on Gradescope.

4 Draft of paper

Your draft should be 4-8 pages. You can talk to me to get approval if there is a good reason for the paper to be longer or shorter. The draft is due on Mar 26. You should write the paper in LaTeX (talk to me if you haven't used LaTeX before), but I will accept any pdf document. Section 9 provides direction about what makes a good math paper and Section 8 gives what your paper will be graded on.

A good resource for learning LaTeX is www.haverford.edu/mathematics-and-statistics/student-resources.

5 Constructive feedback

This step should help you by both providing feedback on your paper and giving you the chance to see what others are doing well or not so well. After you turn in your draft, I will assign other students to read your groups' paper and provide feedback (there will be a feedback form to fill out for each paper you read). You will give constructive criticism, emphasis on constructive, for two other groups' papers. If you make negative comments, are unhelpful, or are mean, I will not give the comments to the other group and you will receive a zero for the feedback portion of the final project grade. The feedback forms will be due Apr 9. I will return the feedback forms along with my own soon after Apr 9. These should be helpful in improving your draft.

6 Final draft of paper

The final draft of your groups' paper will be due on Teams in pdf form on Apr 27 at 3:30 pm. Section 9 provides direction about what makes a good math paper and Section 8 gives what your paper will be graded on.

7 Presentation

Your group will give a 20 minute presentation on your final project. Your audience is the rest of the class. Each person in your group should talk for approximately the same amount of time. Your presentation should be well rehearsed and stick to the time given. Section 11 provides further direction about what makes a good math talk and what you will be graded for.

8 Grading for the paper ¹

Foremost, your paper should be addressed to a fellow Math 4803 student and should present some aspect of your topic in fairly complete detail. Though there could be variations depending on your topic, your paper should include:

- Motivation for why your topic is of interest;
- A statement of the central definitions;
- An explanation of the theorems and/or methods used to answer questions about your topic;
- Proofs of your theorems or major computations; and
- Examples which help clarify your definitions and ones which show your theorem in action.

Your grade on the paper will be based on the following criteria:

The level of your discussion: You are expected to achieve a level of ownership of your topic so that you can communicate it clearly, correctly, and succinctly. Remember that you should think of your audience as another student in Math 4803! What does your audience already know? What does your audience need to know in order to understand your paper? What does your audience not need to know? Why would your audience be interested in your topic? What are appropriate examples to help your audience understand your topic?

The clarity of your writing/presentation: Have you explained your strategy for solving the problem (and not just gone ahead and solved it)? What are the more important steps that you took? What are the less important steps? Is it necessary to articulate all of the less important steps, or can your audience easily follow your reasoning without some of them? You may want to consider putting any particularly nasty calculations in an appendix — or simply citing them after you have understood them — if they are getting in the way of your explanation.

Good grammar and correct spelling are essential. A deficiency in either one detracts from the clarity and force of your writing and will be penalized. In particular, you must write in complete sentences and complete paragraphs. As tempting as it is, you should mostly avoid using technical symbols like \exists , \forall , and \Rightarrow , as they make your prose all but unreadable except in rare circumstances. Equations are part of sentences and should be punctuated as such. I also highly suggest using the imperative and/or the editorial “we” — using “I” or “you” is not acceptable in formal mathematical writing.

The demonstrated depth and correctness of your mathematics: I don’t think that I need to explain this one.

¹Much of this section is from Josh Sabloff.

9 What makes a good math paper?

The following are excerpts from Steven L. Kleiman and Glenn P. Tesler's Writing a Math Phase Two Paper. If you are interested in more details, you can find a copy of the full document here: <http://www.math.uchicago.edu/~may/VIGRE/VIGRE2010/piiUJM2.pdf>.

9.1 Organization

Most short technical papers are divided up into about a half-dozen sections, which are numbered and titled. (The pages too should be numbered for easy reference.) Most papers have an abstract, an introduction, a number of sections of discussion, and a list of references, but no formal table of contents or index. On occasion, papers have appendices, which give special detailed information or provide necessary general background to secondary audiences. Normally, the abstract is three-to-six lines long; the list of references has three-to-nine entries; and each remaining section fills one-to-three pages.

In some fields, papers routinely have a conclusion. This section is not present simply to balance the introduction and to close the paper. Rather, the conclusion discusses the results from an overall perspective, brings together the loose ends, and makes recommendations for further research. In mathematics, these issues are almost always treated in the introduction, where they reach more readers; so a conclusion is rare.

Sectioning involves more than merely dividing up the material; you have to decide what to put where, what to leave out, and what to emphasize. If you make the wrong decisions, you will lose your readers. There is no simple formula for deciding, because the decisions depend heavily on the subject and the audience. However, you must structure your paper in a way that is easy for your readers to follow, and you must emphasize the key results.

The **title** is very important. If it is unclear or misleading, then it will not attract all the intended readers. A strong title identifies the general area of the subject and its most distinctive features. A strong title contains no secondary details and no symbols. A strong title is *concise* — rather short and to the point.

The **abstract** is the most important section. First it identifies the subject; it repeats words and phrases from the title to corroborate a reader's first impression, and it gives details that didn't fit into the title. Then it lays out the central issues, and summarizes the discussion to come. The abstract includes no general background material and preferably no symbols. It just summarizes the contents. The abstract allows readers to decide quickly about reading on. Although many will decide to stop there, the potentially interested will continue. The goal is not to entice all, but to inform the interested efficiently. Remember, readers are busy. They have to decide quickly whether your paper is worth their time. They have to decide whether the subject matter is of interest to them, and whether the presentation will bog them down. A well-written abstract will increase the readership.

The **introduction** is where readers settle into the "story," and often make the final 4 MIT Undergraduate Journal of Mathematics decision about reading the whole paper. Start strong; don't waste words or time. Your readers have just read your title and abstract, and they've gained a general idea of your subject and treatment. However, they are probably still wondering what exactly your subject is and how you'll present it. A strong introduction answers these questions with clarity and precision, but in nontechnical terms. It identifies the subject precisely, and instills interest in it by giving details that did not fit into the title or abstract, such as how the subject arose and where it is headed, how it relates to other subjects and why it is important. A strong introduction touches on all the significant points, and no more. A strong introduction

gives enough background material for understanding the paper as a whole, and no more. Put background material pertinent to a particular section in that section, weaving it unobtrusively into the text. A strong introduction discusses the relevant literature, citing a good survey or two.

Finally, a strong introduction describes the organization of the paper, making explicit references to the section numbers. It summarizes the contents in more detail than the abstract, and it says what can be found in each section. It gives a road map, which indicates the route to be followed and the prominent features along the way. This road map is essentially a table of contents in a paragraph of prose. It is always placed at the end of the introduction to ease the transition into the next section.

The **body** discusses the various aspects of the subject individually. In writing the body, your hardest job is developing a strategy for parceling out the information. Every paper requires its own strategy, which must be worked out by trial and error. There are, however, a few guidelines. First, present the material in small digestible portions. Second, don't jump haphazardly from one detail to another, and don't illogically make some details specific and others generic. Third, try to follow a sequential path through the subject. If such a path doesn't exist, simply break the subject down into logical units, and present them in the order most conducive to understanding. If the units are independent, then order them according to their importance to the primary audience.

There are three main reasons for dividing the body into sections: (1) the division indicates the strategy of your presentation; (2) it allows readers to quickly and easily find the information that interests them; and (3) it gives readers restful white space, allowing them to stop and reflect on what was said. Make the introduction and the several sections of the body roughly equal in length. When you title a section, strive for conciseness, precision, and clarity; then readers will have an easier time jumping to a particular topic. Don't simply insert a title, as is often done in newspaper articles, to recapture interest; rather, wind down the discussion in the first section in preparation for a break, and then restart the discussion in the next section, after the title. When you refer to Section 3, remember to capitalize the word "Section"; it is considered a proper name. Don't subsection a short paper; the breaks would make the flow too choppy.

Accent each main point via stylistic repetition, illustration, or language. Stylistic repetition is the selective repetition of something important; for example, you should talk about the important points once in the abstract, a second time in the introduction, and a third time in the body. When appropriate, repeat an important point in a figure or diagram. Finally, accent an important point with a linguistic device: italics, boldface, or quotation marks; a one-sentence paragraph; or a short sentence at the end of a long paragraph. In particular, set a technical term in italics or boldface—or enclose it in quotation marks if it is only moderately technical—once, at the time it is being defined. Do not underline when italics or boldface is available. Use headings such as **Table 1**, **Figure 2**, and **Theorem 3**, and refer to them as Table 1, Figure 2, and Theorem 3; note that the references are capitalized and set in roman. When you employ linguistic devices, be consistent: always use the same device for the same job.

The list of **references** contains bibliographical information about each source cited. The style of the list is different in technical and nontechnical writing; so is the style of citation. In fact, there are several different styles used in technical writing, but they are relatively minor variations of each other. The style used in this paper is commonly used in contemporary mathematical writing.

The citation is treated somewhat like a parenthetical remark within a sentence, but the reason for the citation must be immediately apparent. Footnotes are not used; neither are the abbreviations "loc. cit.," "op. cit.," and "ibid." The reference key, traditionally a numeral, is enclosed in square brackets.² Within the brackets

²If using BibTeX, this can be accomplished using the `\cite` command.

and after the reference key, place—as a service—specific page numbers, section numbers, or equation numbers, preceded by a comma; see Gillman’s book [5]. The reason for the citation must be immediately apparent, and governs its placement, for example, after a mention of an author’s name or work. If the citation comes at the end of a sentence, put the period after the citation, not before the brackets or inside them. In the list of reference, give the full page numbers of each article appearing in a journal, a proceedings volume, or other collection; do not give the numbers of the particular pages cited in the text.

9.2 Mathematics

Mathematical writing tends to involve many abstract symbols and formal arguments, and they present special problems. To help you understand these problems and deal with them in your writing, here are some comments and guidelines.

Formulas are difficult to read because readers have to stop and work through the meaning of each term. Don’t merely list a sequence of formulas with no discernible goal, but give a running commentary. Define terms as they are introduced, state any assumptions about their validity, and give examples to provide a feeling for them. Similarly, motivate and explain formal statements. Don’t simply call a statement “important,” “interesting,” or “remarkable,” but explain why it is so.

Display an important formula by centering it on a line by itself, and give a reference number in the margin if you need to refer to it. Also display any formula that’s more than a quarter of a line long, that would be broken badly between lines, or that sticks out into the margin. Punctuate the display with commas, a period, and so forth as you would if you had not displayed it; see Section 5 for some examples. Keep in mind that the display is not a figure, but an integral part of the sentence, and therefore needs punctuation.

Be clear about the status of every assertion; indicate whether it is a conjecture, the previous theorem, or the next corollary. If it is not a standard result and you omit its proof, then give a precise reference, in the text just before the statement. Tell whether the omitted proof is hard or easy to help readers decide whether to try to work it out for themselves. If the theorem has a name, use it: say “by the First Fundamental Theorem,” not just “by Theorem 1.” State a theorem before proving it. Keep the statement concise; put definitions and discussion elsewhere.

Prefer a conceptual proof to a computational one; ideas are easier to communicate, understand, and remember. Omit the details of purely routine computations and arguments—ones with no unexpected tricks and no new ideas. Beware of any proof by contradiction; often there’s a simpler direct argument. Finally, when the proof has ended, say so outright. For instance, say, “The proof is now complete,” or use the Halmos symbol \square . In addition, surround the proof—and the statement as well—with some extra white space. (These matters are usually now handled by a LaTeX style file.)

Here are some more guidelines:

1. Separate symbols in different formulas with words.

Bad: Consider S_q , $q = 1, \dots, n$.

Good: Consider S_q for $q = 1, \dots, n$.

2. Don’t use such symbols as \exists , \forall , \wedge , \Rightarrow , \approx , $=$, $>$ in text; replace them by words. They may, of course, be used in formulas placed in text.

Bad: Let S be the set of all numbers of absolute value < 1 .

Good: Let S be the set of all numbers of absolute value less than 1.

Good: Let S be the set of all numbers x such that $|x| < 1$.

3. Don't start a sentence with a symbol.

Bad: $ax^2 + bx + c = 0$ has real roots if $b^2 - 4ac \geq 0$.

Good: The quadratic equation $ax^2 + bx + c = 0$ has real roots if $b^2 - 4ac \geq 0$.

4. Beware of using symbols to convey too much information all at once.

Very bad: If $\Delta = b^2 - 4ac \geq 0$, then the roots are real.

Bad: If $\Delta = b^2 - 4ac$ is nonnegative, then the roots are real.

Good: Set $\Delta = b^2 - 4ac$. If $\Delta \geq 0$, then the roots are real.

5. If you introduce a condition with "if," then introduce the conclusion with "then."

Very bad: If $\Delta \geq 0$, $ax^2 + bx + c = 0$ has real roots.

Bad: If $\Delta \geq 0$, the roots are real.

Good: If $\Delta \geq 0$, then the roots are real.

6. Don't set off by commas any symbol or formula used in text in apposition to a noun.

Bad: If the discriminant, Δ , is nonnegative, then the roots are real.

Good: If the discriminant Δ is nonnegative, then the roots are real.

7. Use consistent notation. Don't say " A_j where $1 \leq j \leq n$ " one place and " A_k where $1 \leq k \leq n$ " another place.

8. Keep the notation simple. For example, don't write " x_i is an element of X " if " x is an element of X " will do.

9. Precede a theorem, algorithm, and the like with a complete sentence.

Bad: We now have the following

Theorem 4. $H(x)$ is continuous.

Good: We can now prove the following result.

Theorem 4. Let $H(x)$ be the function defined by Formula (4). Then $H(x)$ is continuous.

10 Citations

You need to cite all your sources. Remember that it is fine to look at places like Wikipedia to get ideas about your topic, but you *must* go beyond them to the published papers or textbooks when you write your paper.

You should have a reference section at the end of your paper listing the references you cite in your paper in alphabetical order. Here are a few examples of what entries should look like:

Article in a journal:

[1] Y. Eliashberg, *Classification of Overtwisted Contact Structures on 3-Manifolds*. Invent. Math. **98** (1989), 623–637.

Article in an edited volume:

[2] A. Floer, *Monopoles on Asymptotically Flat Manifolds* in the Floer Memorial Volume. Edited by H. Hofer et al. Progress in Mathematics 133. Birkhauser Verlag, Basel, 1995.

Preprint (from arXiv): (Note that you should cite the published version of the article if it has been published.)

[3] L. Ng, *Framed Knot Contact Homology*. Preprint available as arXiv:GT.0407071, 2004.

Book:

[4] M. Schwarz, *Morse Homology*. Birkhauser Verlag, Basel, 1993.

I highly suggest using BibTeX. You can find more information about BibTeX here: www.bibtex.org. BibTeX takes care of formatting and numbering of bibliography entries so that you don't have to update the reference number (such as [1]) in your text if you add a reference earlier in the alphabet. One of the great things about BibTeX, is that for most papers you can get the BibTeX entry for them in the following way: (No trying to figure out what information you need and choosing a type of bibliography; BibTeX takes care of figuring out what information is necessary and what order the information needs to go in.)

1. Go to `library.gatech.edu`.
2. Go to Find Databases in the Research Tools bar on the left.
3. Find the listing for MathSciNet (AMS).
4. Enter some information about the paper such as name(s) of the author(s).
5. Find the entry for the paper. (If you click Find It [Hopf link] GT, you will get a list of ways to get the published paper through the GT library. Frequently you can get pdfs of the published paper.)
6. Under Select alternative format, select BibTeX and just copy that information over into your .bib file. You will likely want to change the label that is given to the BibTeX entry as it will likely be something like MR3624915, which is hard to remember to cite. I generally rename the entry by a list of the author(s) last name(s) without spaces.

You can also find BibTeX entries through Google Scholar, but frequently these have much less information.

11 What makes a good math talk? (And how will you be graded?)³

The 20 minute in-class presentation will have a different flavor, as you have only 20 minutes to convey the main ideas you learned while researching your project. You will not be able to say everything, so you will have to think hard about the most important aspects of your topic and how best to convey them to your classmates. Remember that you are the teacher for this topic — your classmates will not hear it from anyone else! We will discuss what makes a good presentation in class. Each person in your group should spend approximately the same amount of time talking. Also, your presentation really should be 20 minutes,

³This section is also largely from Josh Sabloff.

but make sure you leave some time for questions during your talk. This means you should practice your presentation as a group a bunch of times to make sure you have the timing and transitions down.

I highly suggest making slides, whether Powerpoint, Beamer, etc. If you have a strong preference for giving a chalk talk, come talk to me so we can make sure this suits your project.

The following AMS Blog post has some good suggestions and resources about how to give a good math talk: blogs.ams.org/mathmentoringnetwork/2014/08/04/math-talk-preparing-your-conference-presentation/.

The presentation will be graded based on the first two grading criteria for the paper: level of discussion and clarity of presentation. The following will be taken into account:

1. The mathematics presented should...
 - Be correct
 - Define and use good notation
 - Be elegant (if possible)
2. The presentation of the proof/calculations should include...
 - Clear statements of what is given and what is to be proven/done
 - An outline that “persists” through the proof/calculation
 - A discussion of the ideas behind the proof/calculation for longer and more difficult proofs/calculations; this is not necessary for shorter proofs/calculations
 - References to earlier statements if necessary
 - Pictures if applicable
3. The presenter should...
 - Begin with overview
 - Give context
 - Clearly state main definitions
 - Give credit where necessary
 - Make precise mathematical statements
 - Create and release mathematical tension
 - Slides/handwriting should be legible
 - Speak audibly and at a good pace
 - Make eye contact
 - Express enthusiasm
 - Be in control of subject matter
 - Explain the main difficulties in the work
 - Display confidence
 - Say the right amount
 - End the presentation with purpose
 - Answer questions comfortably

* This document was originally created by Caitlin Leverson and slightly modified for this course.