Scores: 1 2 3 4 5 6 Name Prof. M

Mathematics 4432

Midterm 1

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1. Define *homeomorphism.*

A homeomorphism between two spaces X & Y is a continuous function $f: X \longrightarrow Y$ with continuous inverse.

Define *knot.*

A knot is a simple polygonal curve in $\mathbb{R}^3$.

Define *knot invariant.*

A knot invariant is a function from $\{\text{knots}\}/\sim$ to some set.

2. True or false

Every knot diagram can be changed into a diagram of the trivial knot by changing over-crossings to under-crossings.

T

If A is homeomorphic to A' and B is homeomorphic to B' then $A \times B$ is homeomorphic to $A' \times B'$.

T

The set of irrational numbers in $\mathbb{R}$ has uncountably many path components.

T

Any two knots lying in $\mathbb{R}^2$ are equivalent.

T

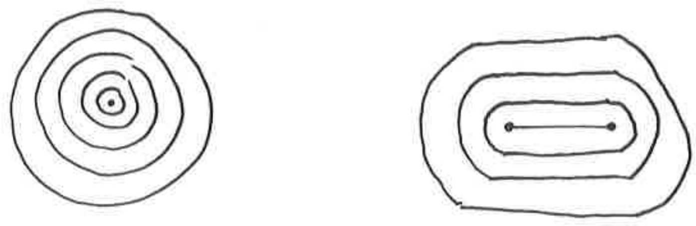
The open unit disk $\{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 < 1\}$ is homeomorphic to $\mathbb{R}^2$.

T

3. Consider the sets

$$A = \{(0,0)\} \text{ and } B = \{(x,0) \mid -1 \le x \le 1\}$$

in $\mathbb{R}^2$. Describe a homeomorphism between $\mathbb{R}^2 \setminus A$ and $\mathbb{R}^2 \setminus B$. *Hint: Consider the set of points lying at distance r from A or B, for varying r.*



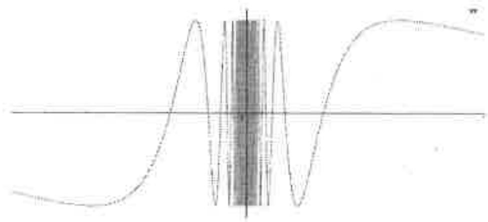
map circles of distance r
to "circles" of distance r
preserving angles.

Are A and B ambient isotopic in $\mathbb{R}^2$? Why or why not?

No. Ambient isotopic implies
homeomorphic.

4. Consider the subset of $\mathbb{R}^2$ given by

$$\{(x, \sin(1/x)) \mid x \in (\infty, 0)\} \cup \{(0, y) \mid -1 \leq y \leq 1\} \cup \{(x, \sin(1/x)) \mid x \in (0, \infty)\}.$$



Prove that this space is not homeomorphic to $\mathbb{R}$.

Let $p(t) = (x(t), y(t))$ be a path with

$x(0) > 0$, $x(1) = 0$

and $x(t) > 0$ $0 \leq t < 1$.

WLOG say $p(1) = (0,0)$.

Let $\varepsilon = 1/2$.

For any δ $x([1-\delta, 1])$ ~~has pts with~~

contains an interval around 0.

$\Longrightarrow y([1-\delta, 1])$ contains 1,

which does not lie in $(-1/2, 1/2) = (0 - \varepsilon, 0 + \varepsilon)$

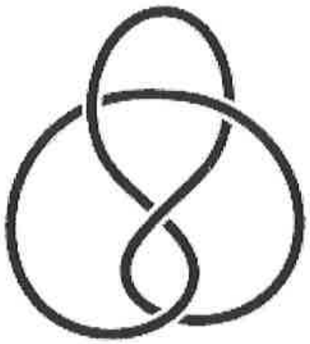
$\Longrightarrow p$ is not continuous.

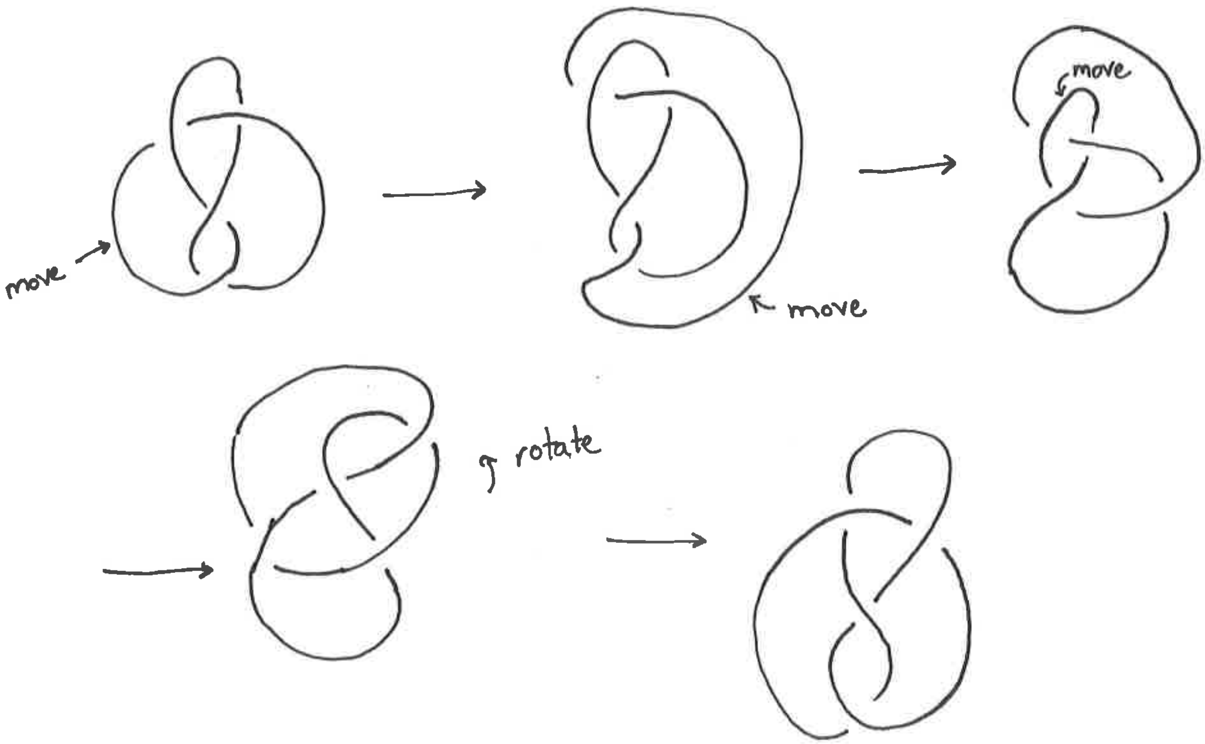
5. Prove that A and Q are not homeomorphic subsets of $\mathbb{R}^2$.

Q has a point that separates it into

3 path components while A does not

6. Show by a sequence of pictures that the Figure 8 knot is equivalent to its mirror image.





How many Reidemeister moves did you use?

8, All in the first arrow